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Response Surface Analysis for Engineering Behavior of Sand-Tire Crumb Mixtures

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Abstract. This paper describes the applicability of concept of Response Surface Methodology (RSM) to identify an approximate response surface model from experimental investigations on the engineering properties of sand and sand-tire crumb mixture (STCM). The objective of the present study is to generate an appropriate polynomial functions for output data obtained from the experimental studies through regression analysis for a range of expected variations in the input parameters. A regression model for both individual and interactive effects of selected input parameters has been generated. It has been found that multiple regression models reproduced the results obtained from the experimental studies with reasonable accuracy. The developed model is also checked for model adequacy and hypothesis testing. None of the developed models revealed any model inadequacy ensuring adequate approximation of the engineering behavior of STCM

Keywords. Hypotheses testing, Model adequacy, Regression model

Introduction

Response Surface Methodology (RSM) is a collection of statistical and mathematical techniques used mostly in the empirical studies [1]. In the practical application of RSM, it is necessary to develop an approximate model for the true response surface. Statistical methods, including statistical process control and design of experiments, play a key role in this activity [2]. RSM is the process of identifying and fitting an approximate response surface model from input and output data obtained from experimental studies or from the numerical analysis where each run can be regarded as an experiment [3].

Materials such as tire crumbs are widely used as light weight materials for backfill in embankment construction due to increased demand for sustainable and environmentally safe disposal of waste materials in civil engineering [4]. Direct shear and unconsolidated undrained (UU) triaxial tests were carried out to evaluate the engineering behaviour of sand and sand-tire crumb mixture (STCM), considering size and percentage of tire crumbs in the mixture. The objective of the present study is to

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identify an approximate response surface model from experimental investigations on the engineering properties of sand and STCM. A regression model for both individual and interactive effects of selected input parameters reproduced the experimental results quite well. The developed regression model is also checked for model adequacy and hypothesis testing to ensure adequate approximation of the system.

1. An experimental program

1.1 Overview

In the present study, engineering behaviour of STCM was evaluated in terms of shear strength, failure strength, secant modulus (modulus of elasticity), ductility and energy absorption capacity through the direct shear and UU test. The features of the above parameters were expressed by measuring the output response variable (y), presented in Table 1. The input variables are converted into coded variables and are presented in Table 2 for both the tests.

Table 1. Response variables of experimental study

Feature investigated	Response variable (y)	Notation	Test carried out
Peak strength	Shear stress	q_u	Direct Shear Test
	Shear strain	SS	
	Deviatory stress at failure	q_f	UU triaxial test
Ultimate	Ultimate deviatory stress	q_{ult}	
Secant modulus	Modulus of elasticity	E	
Ductility	Brittleness index	I_B	
Toughness	Energy absorption capacity	EA	

Table 2. Features investigated and input response variables for regression model

Input Variables	Direct shear test	UU triaxial test
Crumb size (mm)	A(-1); B(-0.57); C(0.63); D(1)	A(-1); B(-0.57); C(0.63); D(1)
Crumb content (%)	0(-1);10(-0.43);15(-0.143); 20(0.143); 25(0.43);30(0.71);35(1)	0(-1);10(-0.43);15(-0.143); 20(0.143); 25(0.43);30(0.71);35(1)
Normal stress / Confining pressure (kPa)	50(-1); 100(0); 150(1)	20(-1); 60(0); 100(1)
Relative density (%)	80	80

- Numbers within brackets represent codified values

A good experimental design provides detailed plans before conducting actual experiments. Experimental planning was oriented by techniques generally known as the design of experiments (DOE) that establishes the necessary combinations of input variables to effectively apply statistical analyses in the sequence [5]. In this study two sets of experiments (direct shear and UU tests) were planned to fit response models through regression analysis in order to observe the effect of input variables (Table 2) on the response variable (y). The experimental design indicated that 75 UU triaxial and direct shear tests were required to assess the interaction of tire crumbs with sand for different crumb sizes and confining pressures (Table 2). The input variables which are also termed as natural variables are converted into codified variables through regression analysis. Coded input variables will have the value of 1 for maximum level, 0 for centre point and -1 for minimum level. Values for other levels are obtained though linear interpolations. The numbers in brackets for each input variable represents the codified values in Table 2.

1.2 Experimental materials and testing procedure

In the present study, locally available sand and industry produced tire scraps are used. The grain size of sand varies between 0.075 mm and 4.75 mm. The specific gravity of sand is 2.65 and is classified as uniformly graded sand. The collected tire crumbs from local industry are sieved into groups of four different sizes, 1 mm - 2mm, 2 mm - 4.75 mm, 4.75 mm – 5.6 mm and 5.6 mm - 8 mm (passing the 8 mm sieve and retained on 5.6 mm). The tire crumbs are labeled as A to D, respectively.

STCM samples were prepared for a constant relative density of 80%. STCM samples has been prepared for 10%, 15%, 20%, 25%, 30% and 35% tire crumbs by volume. The STCM test specimen were prepared in layers with uniform mix, such that segregation would not occur during the sample preparation. Direct shear and static triaxial tests were carried out in accordance with ASTM D5321 and ASTM D2850 standards respectively.

2. Test results and analysis

Shear strength characteristics of the composite materials were examined with respect to the size of tire crumbs, the percentage of tire crumbs and the applied normal stress on the samples in the direct shear tests. Similarly, the effects on energy absorption and deformation characteristics of the composite materials were examined through Unconsolidated Undrained (UU) triaxial tests. The energy absorption capacity (toughness) of the mix were determined by the area traced out by the stress-strain curve obtained from UU tests. The measure of ductility capacity was based on the brittleness index (I_B), which is a function of q_f and q_{ult} and estimated using relation given below:

$$I_B = \frac{q_f}{q_{ult}} - 1$$

where, q_f and q_{ult} are the failure and the ultimate deviatory stress. As the brittleness index decreases towards zero, failure mechanism becomes more ductile.

The regression models employed in RSM are usually quantitative variables obtained from experimental data. The most common empirical models fit the experimental data are either simple linear or more complex quadratic or cubic model. Second order model is widely used in RSM due to its flexibility and considerable practical applications. The equation used to fit a model of response variable (y) with main and interaction effects for n input variables have a general form:

$$y = \beta_0 + \beta_1.x_1 + \beta_2.x_2 + \beta_{11}.x_1^2 + \beta_{12}.x_1.x_2 + \cdots + \beta_n.x_n + \cdots + \beta_{nn}.x_n^2 + \cdots + \beta_{n_1 n_2}.x_{n_1}.x_{n_2}$$

where $x_n = nth$ input variable (here x_1 is crumb size in mm, x_2 is crumb content in percentage, and x_3 is the confining pressure in kPa); β_n =coefficient of nth input variable; and $\beta_0 =$ constant. The equations obtained from regression analysis for different response variables (y) and input variables (Table 1 and 2) are summarized in Table 3. Table 3 shows the observed value of the response variable (y) for the input variables (x_1, x_2, x_3) with the regression coefficients ($\beta_0, \beta_1, \beta_{11}, \dots, \beta_n, \dots, \beta_{nn}$). Above regression equations are valid only over the range of tire crumbs in STCM investigated (for the input codified values). The model adequacy has been checked through the corresponding adjusted coefficient of determination ($R^2_{adjusted}$) are also reported in Table 3.

Table 3. Regression models built using Multiple Regression Analysis

RV (y)	Coefficients of multiple regression equations										
	β_0	$\beta_1.x_1$	$\beta_2.x_2$	$\beta_3.x_3$	$\beta_{11}.x_1^2$	$\beta_{22}.x_2^2$	$\beta_{33}.x_3^2$	$\beta_{12}.x_1.x_2$	$\beta_{13}\,x_1.x_3$	$\beta_{23}.x_2.x_3$	$R^2_{adjusted}$
q_u	68.8	2.2	12.3	35.6	15.8	-13.4	0.4	10.48	3.85	1.46	0.90
SS	7.2	0.12	2.5	0.8	0.9	-0.6	-0.3	0.45	0.06	0.08	0.899
q_f	318.1	31.6	66.1	91.4	29.3	-107.3	7.3	58.37	5.98	-0.20	0.980
q_{ult}	236.6	16.3	73.0	80.6	60.0	-112.9	11.2	54.29	7.27	1.06	0.875
E	8.06	0.4	0.47	1.9	0.4	-0.7	0.01	0.47	0.21	-0.10	0.986
I_B	0.36	0.05	-0.1	-0.1	-0.2	0.14	-0.01	-0.02	0.01	0.01	0.988
EA	48.5	5.2	10.8	15.5	8.3	-14.5	3.2	9.75	4.85	1.10	0.918

Note: RV=Response variables, Regression coefficient equation reported for 3 input variables have the form: $y = \beta_0 + \beta_1.x_1 + \beta_2.x_2 + \beta_3.x_3 + \beta_{11}.x_1^2 + \beta_{22}.x_2^2 + \beta_{33}.x_3^2 + \beta_{12}.x_1.x_2 + \beta_{13}.x_1.x_3 + \beta_{23}.x_2.x_3$

2.1 Direct shear test

The influences of the size of tire crumbs and percentage of tire crumbs on peak shear stress for a normal stress of 150 kPa has been shown in Figure 1. Also, the influence of the percentage of tire crumbs on shear strain corresponding to peak shear stress for different normal stress for crumb size D has been shown in Figure 2. The predicted behavior of the regression model is plotted along with experimental data in both the figures. Regression analysis indicates that the contribution of crumb size to increase peak shear stress of sand is significant for crumb size D, C and B than crumb size A, which is in agreement with Sheikh et al. [6]. The percentage of tire crumbs significantly influences the peak shear stress and peak shear strains for different crumb sizes and confining pressures. Here both the experimental data and regression analysis indicates that the crumb size, percentage of tire crumbs and normal stress influences the shear behavior of STCM. However, the percentage of tire crumbs and normal stress were the main factors that significantly influence shear behavior of STCM. The interactive effect between the variables cannot be found by experimental studies, but regression analysis indicates that the crumb size and crumb content shown to be most significant terms (Table 3). However, the other two interactive terms (crumb size and crumb content, and crumb content and confining pressure) had less effect on peak shear stress and shear strain.

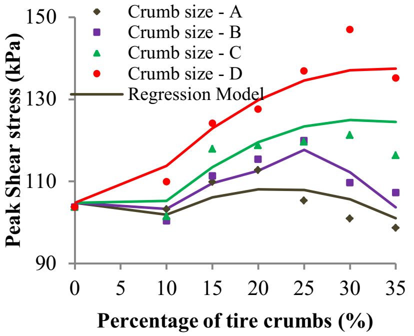

Figure 1. Variation of peak shear stress for normal stress of 150 kPa

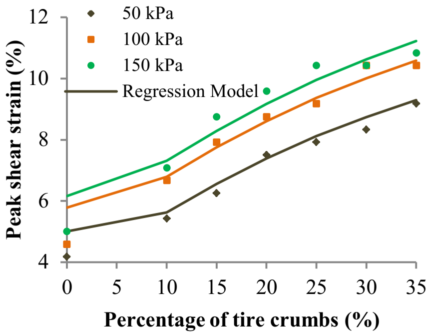

Figure 2. Variation of peak shear strain for crumb size D

2.2 Unconsolidated Undrained test

The same trend is observed from UU test when compared with direct shear test. The regression model showed good agreement with the experimental results. The contribution of each input variable in regression coefficients are presented in Table 3. As expected from direct shear studies, all the three input variables (Crumb size, percentage of tire crumbs and confining pressure) influence the response, but percentage of tire crumbs and confining pressure were the most significant factors which influence deviatoric stress at failure (q_f) and ultimate deviatoric stress (q_{ult}). Crumb size along with the percentage of tire crumbs (first interaction term) was also found to be significant.

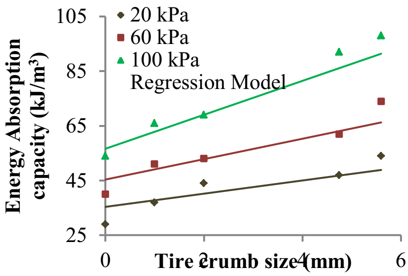

Figure 3. Influence of crumb size on energy absorption capacity

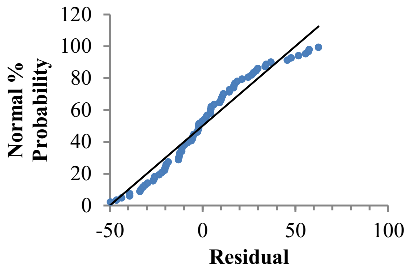

Figure 4. Typical normal probability plot of residuals for q_f

From the regression coefficients presented in Table 3, the influence of confining pressure was found to be the most significant factor. For the brittleness index, the individual parameters (crumb size, crumb content and confining pressure) had significant effect when compared to interaction terms in regression coefficients (Table 3). For each size of tire crumbs failure and ultimate deviatoric stress vary with the percentage of tire crumbs in STCM. For crumb size A, the maximum ultimate shear stress occurs at 20% of tire crumbs by volume in STCM, where as for crumb size B it peaks at 25% of tire crumbs by volume in STCM. For other two crumb sizes (C and D), it peaks at 30% of tire crumbs by volume. With regard to increase in the peak and ultimate strength, the energy absorption capacity which is a measure of area under the stress-strain curve also increases. Figure 3 presents the variation of energy absorption capacity for different crumb sizes and confining pressures, along with the predicted behavior of regression model. As expected, the individual effects of crumb content and confining pressure along with joint effect of crumb size and crumb content were the significant factors which influence on the engineering behavior of STCM. The developed regression model for all the response variables (y) is checked for model adequacy and hypothesis testing to ensure adequate approximation of the engineering behavior of STCM.

3. Hypothesis testing and model adequacy checking

The developed model is also checked for model adequacy through normal probability plots of residuals (typical plot for deviatoric stress at failure is shown in Figure 4), and hypotheses testing through significance of regression and coefficient of multiple determinations to ensure adequate approximation of the system. The coefficient of multiple determinations for the entire fitted regression model showed better variability. None of the developed regression models in this study reveal any model inadequacy and also ensured adequate approximation of the system.

4. Conclusion

In this study the experimental study was carried on STCM to assess the influence of both individual and interactive effects of selected input parameters by RSM. The regression models reproduced the results obtained from both UU and direct shear tests quite well. The predicted behavior of the regression model is plotted along with experimental data. The regression model showed all the individual parameter are sensitive to input variables, but the interactive effect between crumb size and crumb content shown to be most significant interactive terms, which significantly differ from other parameters. The adjusted coefficient of determination explains the variability in the response variables in the fitted regression model.

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